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Faculty of Mathematics and Natural Sciences,
Andalas University, Indonesia

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Maiyastri, Susila Bahri

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Preface

First of all, I would like to say welcome to Bukittinggi, Indonesia to all of you. It is an honour for us to host this conference. We are very happy and proud because the participants of this conference come from many countries; we have participants from Libya, Japan, Qatar, India, Malaysia, Singapore, Thailand, Iran, and many more.

Ladies and gentlemen, according to constructivism theory, mathematics comes out as a result of social construction; that's why, the outcomes of our researches in mathematics, like theorem or formula of mathematics, should be communicated in a scientific forum such as seminar or conference. Through this kind of seminar or conference, we could refine the existing theorems or we could get new ideas to produce a new one. Seminar or conference which is held annually enables us to continually develop the science of mathematics until the end of the time.

That's way, in this two-day conference, we are going to discuss around 250 papers coming from diverse aspects of mathematics ranging from analysis, applied mathematics, statistics, algebra, Computational Mathematics, mathematics education, and other related topics.

For all of us here, I would like to convey my endless appreciation and gratitude for your participation in this conference.

Thank you very much

Dr. I Made Arnawa
Chairman of the Conference

Message from Rector Andalas University

It gives me great pleasure to extend my sincere and warm welcome to the participants of the 5th International Conference on Mathematics Statistics and Application (The IMT GT's 5th ICMSA 2009) - A Joint Scientific Program organized by Universities over Indonesia, Malaysia and Thailand Growth Triangle Region. On behalf of Andalas University, let me welcome all of you to the conference in Bukittinggi, West Sumatra Province, the land of Minang kabau.

We believe that from this scientific meeting, all of participants will have time to discuss and exchange ideas, findings, creating new networking as well as strengthen the existing collaboration in the respective fields of expertise. In the century in which the information is spreading in a tremendous speed and globalization is a trend, Andalas University must prepare for the tough competition that lay a head. One way to succeed is by initiating and developing collaborative work with many partners from all over the world. Through the collaboration in this conference we can improve the quality of our researches as well as teaching and learning process in mathematics and to achieve standards and requirements applied in many developed countries. I strongly believe that this conference is and extraordinary testimony to our capacity building at international, regional and local level.

I would like to express my deep gratitude to International Scientific Committee of who has honored the Mathematics Department, Faculty of Mathematics and Natural Sciences, Andalas University to host this prestigious conference. This is a very special opportunity for us to be involved in a regional community of knowledgeable scientist in the field of mathematics, statistics and their applications. I would also like to extend my gratitude to keynote speakers, participants, and organizer of this conference for their contribution to this event. My special thank is also rendered to the local government of West Sumatra for various supports and facilities.

Finally I wish all participants a fruitful deliberation at the conference. I also wish all participants and accompanying spouses a pleasant and enjoyable stay in Bukittinggi City, West Sumatra.

Prof. Dr. Ir. Musliar Kasim, MS
Rector

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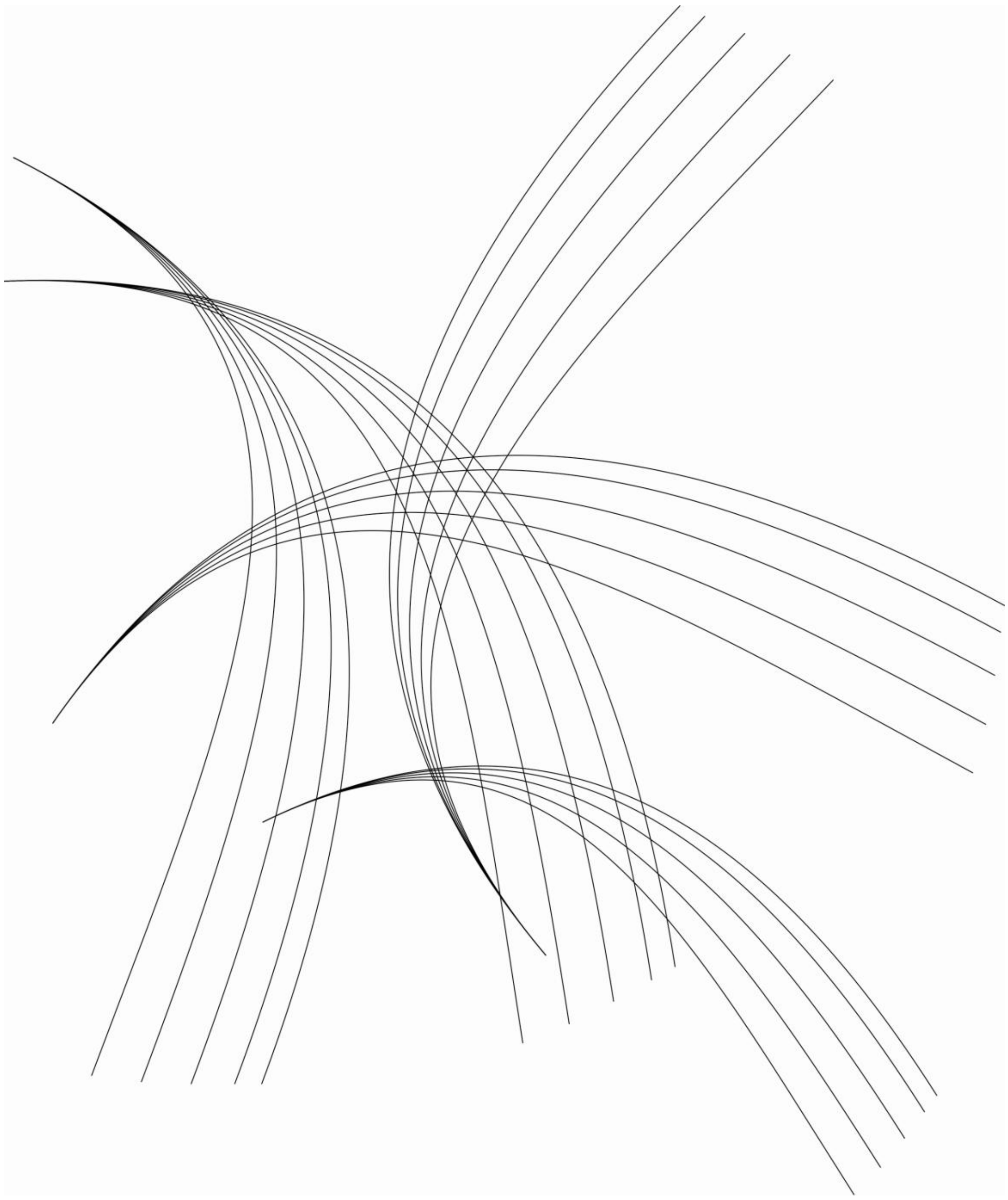
Contents



CONTENTS

Applied Mathematics **445 - 979**

Distribution-free Test for Stability of Run-off Triangle

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Abstract

The estimation of outstanding claims liability (OCL) in long-tail insurance business is often based on run-off triangles. There are several methods to estimate OCL based on run-off triangles which assume a stable pattern of payments by development period within each accident period. These methods are run-off techniques. The chain ladder method is probably the most popular one of the run-off techniques. It is important to test the assumption of the stability of the payments pattern. This paper offers a distribution-free approach to test that stability. The approach is motivated by the basic chain ladder assumption and two additional assumptions in the paper by Mack (1993). In this paper, all link ratios in each development are tested if they are not statistically different from its development factor. For each development period, if at least one of the link ratios is statistically different from its development factor, then we conclude that the payment pattern is unstable. If at least one of the development periods is not stable, then we conclude that the run-off triangle is unstable. The critical values of the test are computed using simulation. Two run-off triangle data are used to illustrate the approach.

Keywords: chain ladder method, development factor, link ratio, outstanding claims liability, run-off triangle.

1. Introduction

In general insurance business, an insurance company must estimate its liability to make future payments for claims that have arisen on or before the valuation date. This liability is called outstanding claims liability (OCL). An insurance company needs to estimate OCL because there is delay between the occurrence of claim and the time of claim is settled.

Statistical methods are usually used to estimate OCL because of the uncertainty of OCL, since the amount and timing of future claims are unknown. The degree of uncertainty depends on whether the insurance business is short-tail or long-tail business (Olofsson, 2006). A short-tail insurance business is when the delay between the occurrence of a claim and the settlement is short, often less than a year, such as automobile physical damage insurance, fire insurance, and earthquake insurance. A long-tail business is an insurance business where the delay between the occurrence of a claim and the settlement is long, more than a year, such as third party liability, marine insurance, reinsurance, medical malpractice, and worker compensation.

It is common to estimate OCL for long-tail business based on run-off triangle data (for example, see Mack, 1993; England, and Verrall, 2002; Verrall, 2002; Panning, 2004; de Jong, 2006; Pinheiro et al., 2006; and Olofsson, 2006). Run-off triangle data provides a summary of underlying data set with individual claim figures (Antonio et al., 2006). There are several methods to estimate OCL based on run-off triangle data that assume a stable pattern of payments by development period within each accident period. These methods called run-off techniques (Hart et al., 1996). The chain ladder method is probably the most popular one of the run-off techniques for estimating OCL. The main reason for this is its simplicity and the fact that it is distribution-free (Mack, 1993). It is frequently used as a gold standard (benchmark) because of its generalized use and easy to apply (de Alba, 2006).

It is important to test the assumption of stability of the payments pattern. If we use run-off techniques to estimate OCL based on unstable run-off data, then the estimation result may be inappropriate. This paper offers a distribution-free approach to test that stability. This approach is motivated by the basic chain ladder assumption and two additional assumptions in the paper by Mack (1993). In this paper, all link ratios in each development are tested if they are statistically equal to its development factor or not. For each development period, if at least one of the link ratios is not statistically equal to its development factor, then we conclude that the payment pattern is unstable. If at least one of the development periods is not stable, then we conclude that the run-off triangle is unstable. The critical values of the test are computed using simulation.

The remainder of the paper is structured as follows. Section 2 presents the chain ladder method. The stability test is discussed in Section 3. The algorithm to compute the critical values of the test using simulation is given in Section 4. Section 5 provides two numerical examples to illustrate the approach.

2. The Chain Ladder Method

Let D_{ij} represent the incremental claim amounts (or the number of claims) in the development period j (j = 1, 2, …, n), corresponding to the accident period i (i = 1, 2, …, n). The cumulative claims are defined by

$$C_{ij} = \sum_{t=1}^{j} D_{it}\,; \quad i = 1, 2, \cdots, n.$$

We consider C_{ij} are random variables of which we have observations if $i + j = 2, 3, \ldots, n + 1$ (run-off triangle). The pattern of the available data can be seen in Table 1.

Table 1
Run-off Triangle Data

Accident Period	***Development Period***						
	1	2	…	j	…	$n-1$	n
1	C_{11}	C_{12}	…	C_{1j}	…	$C_{1,n-1}$	C_{1n}
2	C_{21}	C_{22}	…	C_{2j}	…	$C_{2,n-1}$	
⋮	⋮	⋮	⋮	⋮	⋰		
i	C_{i1}	C_{i2}	…	C_{ij}			
⋮	⋮	⋮	⋰				
$n-1$	$C_{n-1,1}$	$C_{n-1,2}$					
n	$C_{n,1}$						

From now on, and without loss of generality, we consider that C_{ij} are the cumulative claims amounts. The aim of the run-off techniques is to estimate all the empty cells in Table 1 to obtain the estimate of OCL. The most popular one of the run-off techniques for estimating OCL is the chain ladder method (Mack, 1993). The chain ladder method has three assumptions as follows.

(1) There are development factors $\lambda_1, \lambda_2, \cdots, \lambda_{n-1} > 0$ with expectation $E\left(C_{i,j+1} | C_{i1}, \cdots, C_{ij}\right) = C_{ij}\,\lambda_j, i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n-1.$

(2) $\{C_{i1}, \cdots, C_{in}\}, \{C_{k1}, \cdots, C_{kn}\}, i \neq k$, are independent.

(3) Variance $Var\left(C_{i,j+1} | C_{i1}, \cdots, C_{ij}\right) = C_{ij}\,\sigma_j^2, i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n-1,$ with unknown parameters $\sigma_j^2, j = 1, 2, \ldots, n-1.$

The first assumption is the basic chain ladder assumption and the last two assumptions are the additional assumptions stated by Mack (1993). The estimates of the development factors are

$$\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}}, \quad j = 1, 2, \ldots, n-1, \tag{1}$$

and the estimates of parameters $\sigma_j^2, j = 1, 2, \ldots, n-2$ are

$$\hat{\sigma}_j^2 = \frac{1}{n-j-1} \sum_{i=1}^{n-j} C_{ij} \left(\frac{C_{i,j+1}}{C_{ij}} - \hat{\lambda}_j\right)^2, j = 1, 2, \ldots, n-2. \tag{2}$$

Mack (1993) suggested that if $\hat{\lambda}_{n-1} = 1$ and if the claims development is believed to be finished after $n - 1$ years, then $\hat{\sigma}_{n-1} = 0$, otherwise

$$\hat{\sigma}_{n-1}^2 = min\left(\frac{\hat{\sigma}_{n-2}^4}{\hat{\sigma}_{n-3}^2}, min(\hat{\sigma}_{n-3}^2, \hat{\sigma}_{n-2}^2)\right). \tag{3}$$

Theorem 1: *Let $D = \{C_{ij} | i + j = 2, \ldots, n+1\}$ be the data set are observed. Under the assumptions (1) and (2) we have*

$$E(C_{in} | D) = C_{i,n+1-i}\lambda_{n+1-i} \cdot \ldots \cdot \lambda_{n-1}.$$

Theorem 2: *Under the assumptions (1) and (2) the estimators $\hat{\lambda}_j, j = 1, 2, \ldots, n-1$, are unbiased and uncorrelated.*

Theorems 1 and 2 have been proven by Mack (1993), and are used to estimate OCL in the chain ladder method.

Theorem 3: *Under the assumptions (1), (2), and (3) the estimators $\hat{\sigma}_j^2, j = 1, 2, \ldots, n-2$, are unbiased.*

Proof: Begin with the following result, which will be used later:

$$\begin{aligned}\sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\hat{\lambda}_j\right)^2 &= \sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\lambda_j+\lambda_j-\hat{\lambda}_j\right)^2\\ &= \sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\lambda_j\right)^2+2\sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\lambda_j\right)(\lambda_j-\hat{\lambda}_j)+\sum_{i=1}^{n-j} C_{ij}(\lambda_j-\hat{\lambda}_j)^2\\ &= \sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\lambda_j\right)^2+2(\lambda_j-\hat{\lambda}_j)\sum_{i=1}^{n-j}(C_{i,j+1}-C_{ij}\lambda_j)+(\lambda_j-\hat{\lambda}_j)^2\sum_{i=1}^{n-j} C_{ij}\\ &= \sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\lambda_j\right)^2+2(\lambda_j-\hat{\lambda}_j)(\hat{\lambda}_j-\lambda_j)\sum_{i=1}^{n-j} C_{ij}+(\lambda_j-\hat{\lambda}_j)^2\sum_{i=1}^{n-j} C_{ij}\\ &= \sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\lambda_j\right)^2-(\hat{\lambda}_j-\lambda_j)^2\sum_{i=1}^{n-j} C_{ij}. \qquad (*)\end{aligned}$$

We also have (from independence of $\{C_{i1},\cdots,C_{in}\},\{C_{k1},\cdots,C_{kn}\}, i\neq k$)

$$Var(\hat{\lambda}_j)=Var\left(\frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}}\right)=\frac{1}{\left(\sum_{i=1}^{n-j} C_{i,j}\right)^2}\sum_{i=1}^{n-j} Var(C_{i,j+1})=\frac{\sigma_j^2}{\left(\sum_{i=1}^{n-j} C_{i,j}\right)^2}\sum_{i=1}^{n-j} C_{i,j}=\frac{\sigma_j^2}{\sum_{i=1}^{n-j} C_{i,j}}.$$

Take expectation in (*) to obtain

$$\begin{aligned}E\left[\sum_{i=1}^{n-j} C_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\hat{\lambda}_j\right)^2\right] &= \sum_{i=1}^{n-j} C_{ij}\,E\left[\left(\frac{C_{i,j+1}}{C_{ij}}-\lambda_j\right)^2\right]-\sum_{i=1}^{n-j} C_{ij}\,E\left[(\hat{\lambda}_j-\lambda_j)^2\right]\\ &= \sum_{i=1}^{n-j} C_{ij}\,Var\left(\frac{C_{i,j+1}}{C_{ij}}\right)-\sum_{i=1}^{n-j} C_{ij}\,Var(\hat{\lambda}_j)=\sum_{i=1}^{n-j} C_{ij}\frac{1}{(C_{ij})^2}C_{ij}\sigma_j^2-\sum_{i=1}^{n-j} C_{ij}\frac{\sigma_j^2}{\sum_{i=1}^{n-j} C_{ij}}\\ &= (n-j-1)\sigma_j^2.\end{aligned}$$

Dividing both sides by $(n-j-1)$ demonstrates that $\hat{\sigma}_j^2$ is an unbiased estimator of σ_j^2.

3. The Stability Test

Run-off techniques to estimate OCL assume a stable pattern of payments by development period within each accident period. This section discusses a distribution-free approach to test that stability. This approach is motivated by the basic chain ladder assumption and two additional assumptions in the paper by Mack (1993).

The test will be based on the following theorem.

Theorem 4: *Let*

$$W_{ij}=\frac{C_{i,j+1}}{C_{ij}}-\hat{\lambda}_j,\quad j=1,2,\dots,n-1, i=1,\dots,n-j$$

Under the assumptions (1), (2), and (3) we have expectations and variances

$$E(W_{ij}\,|C_{i1},\cdots,C_{ij})=0,$$
$$Var(W_{ij}\,|C_{i1},\cdots,C_{ij})=\left(\frac{1}{C_{ij}}-\frac{1}{\sum_{k=1}^{n-j} C_{kj}}\right)\sigma_j^2.$$

Proof: We use the abbreviations

$$E_{ij}(X)=E(X|C_{i1},\cdots,C_{ij}),$$
$$Var_{ij}(X)=Var(X|C_{i1},\cdots,C_{ij}).$$

For the expectations

$$E_{ij}(W_{ij})=E_{ij}\left(\frac{C_{i,j+1}}{C_{ij}}-\hat{\lambda}_j\right)=\frac{E_{ij}(C_{i,j+1})}{C_{ij}}-E_{ij}(\hat{\lambda}_j)=\frac{C_{ij}\lambda_j}{C_{ij}}-\lambda_j=0.$$

For the variances

$$Var_{ij}(W_{ij}) = Var_{ij}\left(\frac{C_{i,j+1}}{C_{ij}} - \hat{\lambda}_j\right) = Var_{ij}\left(\frac{C_{i,j+1}}{C_{ij}} - \frac{\sum_{k=1}^{n-j} C_{k,j+1}}{\sum_{k=1}^{n-j} C_{kj}}\right)$$

$$= Var_{ij}\left(\frac{C_{i,j+1}}{C_{ij}} - \frac{C_{i,j+1}}{\sum_{k=1}^{n-j} C_{kj}} - \frac{\sum_{k=1, k\neq i}^{n-j} C_{k,j+1}}{\sum_{k=1}^{n-j} C_{kj}}\right)$$

$$= Var_{ij}\left(\left[\frac{1}{C_{ij}} - \frac{1}{\sum_{k=1}^{n-j} C_{kj}}\right] C_{i,j+1} - \frac{\sum_{k=1, k\neq i}^{n-j} C_{k,j+1}}{\sum_{k=1}^{n-j} C_{kj}}\right)$$

$$= Var_{ij}\left(\left[\frac{1}{C_{ij}} - \frac{1}{\sum_{k=1}^{n-j} C_{kj}}\right] C_{i,j+1}\right) + Var_{ij}\left(\frac{\sum_{k=1, k\neq i}^{n-j} C_{k,j+1}}{\sum_{k=1}^{n-j} C_{kj}}\right)$$

$$= \left[\frac{1}{C_{ij}} - \frac{1}{\sum_{k=1}^{n-j} C_{kj}}\right]^2 Var_{ij}(C_{i,j+1}) + \frac{1}{\left(\sum_{k=1}^{n-j} C_{kj}\right)^2} \sum_{\substack{k=1\\k\neq i}}^{n-j} Var_{ij}(C_{k,j+1})$$

$$= \left[\frac{1}{C_{ij}} - \frac{1}{\sum_{k=1}^{n-j} C_{kj}}\right]^2 C_{ij}\sigma_j^2 + \frac{1}{\left(\sum_{k=1}^{n-j} C_{kj}\right)^2} \sum_{\substack{k=1\\k\neq i}}^{n-j} C_{kj}\sigma_j^2$$

$$= \left(\frac{1}{C_{ij}} + \frac{C_{ij}}{\left(\sum_{k=1}^{n-j} C_{kj}\right)^2} - \frac{2}{\sum_{k=1}^{n-j} C_{kj}} + \frac{\sum_{k=1, k\neq i}^{n-j} C_{kj}}{\left(\sum_{k=1}^{n-j} C_{kj}\right)^2}\right)\sigma_j^2 = \left(\frac{1}{C_{ij}} - \frac{1}{\sum_{k=1}^{n-j} C_{kj}}\right)\sigma_j^2.$$

Define the link ratios

$$R_{ij} = \frac{C_{i,j+1}}{C_{ij}}, \quad j = 1, 2, \dots, n-1, i = 1, \dots, n-j. \tag{4}$$

Run-off triangle is stable if all payments patterns in every development are stable. In other words, Run-off triangle is stable if all link ratios in every development are stable. For a given development j (j = 1, 2, …, n – 1), all link ratios are stable if all the difference between link ratio R_{ij} and λ_j are small with high probability or all the difference between link ratio R_{ij} and $\hat{\lambda}_j$ are small relative to standard deviation of W_{ij}, $\sigma_{W_{ij}}$ with high probability. In statistical term, this means that probability

$$P\left(\left|R_{ij} - \hat{\lambda}_j\right| \leq \varepsilon_j \sigma_{W_{ij}}\right) \geq p,$$

or

$$P\left(\left|\frac{W_{ij}}{\sigma_{W_{ij}}}\right| \leq \varepsilon_j\right) \geq p.$$

Because W_{ij} has a continuous distribution, the "≥" sign in the above equation may be replaced by an "=" sign, so

$$P\left(\left|\frac{W_{ij}}{\sigma_{W_{ij}}}\right| \leq \varepsilon_j\right) = p. \tag{5}$$

Equation (5) will be used to test the stability of the run-off triangle. The detailed steps of the stability test of the run-off triangle are as follow

(1) All link ratios in each development period are tested if they are not statistically different from its development factor. For development j (j = 1, …, n – 1), the test statistic is

$$\frac{W_{ij}}{\sigma_{W_{ij}}}, \quad j = 1, \cdots, n-1, i = 1, \cdots, n-j, \tag{6}$$

where

$$W_{ij} = \frac{C_{i,j+1}}{C_{ij}} - \hat{\lambda}_j, \quad j = 1, 2, \dots, n-1, i = 1, \dots, n-j, \tag{7}$$

$\sigma_{W_{ij}}$ is estimated by

$$\hat{\sigma}_{W_{ij}} = \sqrt{\left(\frac{1}{C_{ij}} - \frac{1}{\sum_{k=1}^{n-j} C_{kj}}\right)\hat{\sigma}_j^2}, \quad j = 1, 2, \dots, n-1, i = 1, \dots, n-j. \quad (8)$$

Algorithm to compute the critical values of the test, i.e. ε_j, is discussed in the next section. If the absolute value of the test statistic is less than or equal to the critical value, then we conclude that the link ratio is not statistically different from its development factor.

(2) For each development period, if at least one of the link ratios is statistically different from its development factor, then we conclude that the payment pattern is unstable.

(3) If at least one of the development periods is not stable, then we conclude that the run-off triangle is unstable.

4. The Critical Value of the Test

This section discusses the computation of the critical values of the stability test of run-off triangle simulation. The algorithm to compute critical value for development j is given as follows

(1) Compute the estimated development factor $\hat{\lambda}_j$ using Equation (1).

(2) Compute the estimated variance component $\hat{\sigma}_j^2$ using Equation (2) and (3).

(3) Let C_{ij} $\{i = 1, \cdots, n+1-j\}$ represent the cumulative claims in development j. Compute the estimated mean and the estimated variance of $C_{i,j+1}$ $\{i = 1, \cdots, n-j\}$. The estimated means are $C_{ij}\,\hat{\lambda}_j,\ i = 1, \dots, n-j$, and the estimated variances are $C_{ij}\,\hat{\sigma}_j^2, i = 1, \dots, n-j$.

(4) Generate the cumulative claims $C_{i,j+1}$ $\{i = 1, \cdots, n-j\}$ from a distribution (for example lognormal, gamma, or Weibull) with mean $C_{ij}\,\hat{\lambda}_j,\ i = 1, \dots, n-j$ and variance $C_{ij}\,\hat{\sigma}_j^2, i = 1, \dots, n-j$.

(5) Based on $C_{i,j+1}$ $\{i = 1, \cdots, n-j\}$ obtained from Step (4) and $C_{i,j}$ $\{i = 1, \cdots, n+1-j\}$ from the run-off triangle data, compute

(5.1) The estimated development factor $\hat{\lambda}_j^*$ using Equation (1),

(5.2) The estimated variance component $\hat{\sigma}_j^{2*}$ using Equation (2) and (3),

(5.3) W_{ij}; $i = 1, \dots, n-j$, using Equation (7).

(5.4) $\hat{\sigma}_{W_{ij}}$; $i = 1, \dots, n-j$, using Equation (8).

(5.5) The absolute values of $W_{ij}/\hat{\sigma}_{W_{ij}}$.

(6) Repeat Step (4) and (5) N times.

(7) For a given ε ($\varepsilon = 1, \cdots, 2.5$; with increment 0.01), compute

$$S_i = \#\{|W_{ij}/\hat{\sigma}_{W_{ij}}| \le \varepsilon\}/N, i = 1, \dots, n-j$$

(8) Compute mean of $S_i, i = 1, \dots, n-j$,

$$S = \frac{\sum_{i=1}^{n-j} S_i}{n-j}$$

(9) Repeat Step (4) until Step (8) M times, then compute mean of S.

(10) The critical value of the test is ε such that $S = 1 - \alpha$, where α is the level of significance for the test.

5. The Numerical Examples

This section discusses the implementation of the stability test of run-off triangle for two data sets. The data set 1 are taken from Mack (1993) which were used by Verrall (1990). The data set 1 in Table 2 contain cumulative claims. The data set 2 are taken from de Jong (2006) in Table 3 relates to Automatic Facultative General Liability from the historical Loss Development Study. This triangle, called the AFG data that contain cumulative incurred claim amounts.

Table 2

Data Set 1 - Run-off Triangle Data (Cumulative Figures)

i	C_{i1}	C_{i2}	C_{i3}	C_{i4}	C_{i5}	C_{i6}	C_{i7}	C_{i8}	C_{i9}	C_{i10}
1	357848	1124788	1735330	2218270	2745596	3319994	3466336	3606286	3833515	3901463
2	352118	1236139	2170033	3353322	3799067	4120063	4647867	4914039	5339085	
3	290507	1292306	2218525	3235179	3985995	4132918	4628910	4909315		
4	310608	1418858	2195047	3757447	4029929	4381982	4588268			
5	443160	1136350	2128333	2897821	3402672	3873311				
6	396132	1333217	2180715	2985752	3691712					
7	440832	1288463	2419861	3483130						

8	359480	1421128	2864498							
9	376686	1363294								
10	344014									

Table 3
Data Set 2 – AFG Data in Cumulative

i	C_{i1}	C_{i2}	C_{i3}	C_{i4}	C_{i5}	C_{i6}	C_{i7}	C_{i8}	C_{i9}	C_{i10}
1	5012	8269	10907	11805	13539	16181	18009	18608	18662	18834
2	106	4285	5396	10666	13782	15599	15496	16169	16704	
3	3410	8992	13873	16141	18735	22214	22863	23466		
4	5655	11555	15766	21266	23425	26083	27067			
5	1092	9565	15836	22169	25955	26180				
6	1513	6445	11702	12935	15852					
7	557	4020	10946	12314						
8	1351	6947	13112							
9	3133	5395								
10	2063									

Table 4 and 5 contains the test statistics and the critical values for data set 1 and data set 2, respectively. The run-off triangle for data set 1 is stable because in Table 4, all link ratios are not statistically different from its development factor (for each development, all test statistics is less than its critical value). However, the run-off triangle for data set 2 is unstable because in Table 5, the development periods 1, 2, 3, and 5 are not stable.

Table 4
Test Statistics and Critical Values for Data Set 1

i	Development							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0,183	0,144	0,131	0,049	0,085	0,036	0,011	0,011
2	0,010	0,006	0,073	0,040	0,019	0,041	0,004	0,011
3	0,448	0,023	0,001	0,055	0,068	0,033	0,007	
4	0,523	0,161	0,213	0,106	0,017	0,040		
5	0,550	0,089	0,079	0,000	0,032			
6	0,070	0,086	0,074	0,057				
7	0,336	0,099	0,016					
8	0,244	0,215						
9	0,070							
Critical Values	**2,03**	**2,1**	**2,08**	**2,02**	**1,9**	**1,7**	**> 1,41**	**> 0,99**

Table 5
Test Statistics and Critical Values for Data Set 2

i	Development							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	0,652	0,896	0,803	0,368	0,940	1,583	0,000	1,000
2	2,313*	0,743	2,038*	1,688	0,214	1,057	1,282	1,000
3	0,138	0,249	0,526	0,195	1,010	0,349	1,166	
4	0,500	0,931	0,413	1,484	0,001	0,130		
5	1,169	0,103	0,685	0,019	1,809*			
6	0,304	0,490	0,734	0,842				
7	0,604	2,167*	0,622					
8	0,487	0,703						
9	0,463							
Critical Values	**2,28**	**1,9**	**1,87**	**1,82**	**1,76**	**1,65**	**1,41**	**> 1,01**

* means significance at $\alpha = 5\%$.

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