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*Prosiding*

## **Seminar Nasional Mahasiswa S3 Matematika**

# **REVITALISASI DAN SOSIALISASI DIRI UNTUK BERPERAN AKTIF DALAM PENINGKATAN KUALITAS PENELITIAN & PENDIDIKAN MATEMATIKA DI INDONESIA**

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Forum Komunikasi Mahasiswa S-3 Matematika se-Indonesia  
Bekerja Sama Dengan Jurusan Matematika FMIPA UGM

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# **PROSIDING SEMINAR NASIONAL MAHASISWA S3 MATEMATIKA INDONESIA 2008**

**EDITOR:**

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## KATA PENGANTAR

Puji syukur kami panjatkan kehadiran Allah SWT atas terselenggarakannya Seminar Nasional Mahasiswa S3 Matematika Indonesia pada tanggal 31 Mei 2008 di Jurusan Matematika FMIPA UGM Yogyakarta. Kegiatan ini merupakan kegiatan ilmiah rutin yang diadakan oleh Forum Komunikasi Mahasiswa S3 Matematika Indonesia sejak tahun 2005.

Seminar Nasional ini diselenggarakan dengan tujuan (1) memberikan wawasan dan berbagi informasi diantara mahasiswa matematika/statistika dan pendidikan matematika, atau peminat matematika tentang hasil penelitian yang telah atau sedang dilakukan; (2) mendorong terciptanya kerjasama keilmuan antara mahasiswa S3 matematika/statistika dan pendidikan matematika, atau antara mahasiswa S3 matematika/statistika/pendidikan matematika dengan para pakar matematika/statistika/pendidikan matematika lainnya di Indonesia; dan (3) membentuk korps keilmuan untuk pengembangan profesi ke depan dan menjalin silaturahmi antara sesama mahasiswa dan alumni S3 matematika/statistika/pendidikan matematika se-Indonesia.

Peserta yang berpartisipasi dalam kegiatan ini berjumlah 90 orang yang berasal dari berbagai institusi/instansi, yaitu ITB, UNY, UPI, UT, UNMER Malang, UNMUL, UNNES, UM, IPB, UNIMED, USD, ITS, UNSOED, UNTAD, IAIN Antasari Banjarsari, UNISBA, Universitas Widya Dharma Klaten, UNESA, SMA Sang Timur Yogyakarta, UBAYA, Universitas Veteran Bangun Nusantara Sukoharjo, UNIKA Parahyangan, UNTAD, Universitas Mahasaraswati, UI, UII, UNPAD, STAIN Purwokerto, UNHALU, UNSRAT, Universitas PGRI Palembang, UNCEN, UNPAD, STKIP YASIKA Majalengka, UNEJ, STKIP PGRI Pontianak, UIN Syarif Hidayatullah, UNPATI, STT Garut, UNILA, Universitas Negeri Gorontalo, SMKN 2 Yogyakarta, BATAN, UBINUS, Universitas Muhammadiyah Bengkulu, UNAND, SKIP Siliwangi Bandung, Sekolah Tinggi Sandi Negara, dan UGM. Di antara peserta ini, yang membawakan makalah berjumlah 50 orang. Pembicara utama pada seminar ini adalah Prof. Dr. Muchlas Samani (Direktur Ketenagaan Ditjen Dikti Depdiknas) dan Dr. Supama, M.Si. (Peneliti Bidang Analisis Jurusan Matematika FMIPA Universitas Gadjah Mada).

Terselenggaranya Seminar Nasional ini tidak lepas dari dukungan berbagai pihak. Untuk itu terima kasih kami sampaikan kepada peserta Seminar Nasional atas partisipasinya yang aktif serta kepada Fakultas MIPA UGM dan Jurusan Matematika FMIPA UGM atas fasilitas dan dukungan dananya. Ucapan terima kasih juga kami sampaikan kepada Forum Komunikasi Mahasiswa S3 Matematika Indonesia atas dukungan dan publikasinya. Terakhir, terima kasih juga kami sampaikan kepada rekan-rekan Panitia dan staf Jurusan Matematika FMIPA UGM atas dukungan dan kerjasamanya sehingga Seminar Nasional ini dapat terselenggara dengan lancar.

Yogyakarta, Mei 2008

Ketua Panitia

**SAMBUTAN :  
PENGELOLA S2/S3 MATEMATIKA FMIPA UGM**

Yang terhormat Dekan Fakultas MIPA,  
Yang terhormat Direktur Ketenagaan Ditjen Dikti Depdiknas,  
Yang terhormat Staf Pengajar S3 Matematika FMIPA UGM,  
Yang terhormat Hadirin Peserta Seminar Nasional,  
*Assalamu 'alaikum Warahmatullaahi Wabarakaatuh,*

Selamat pagi dan salam sejahtera bagi kita semua.

Pertama-tama marilah kita memanjatkan puji dan syukur kehadiran Tuhan Yang Maha Kuasa karena atas perkenan-Nya kita dapat berkumpul pada acara seminar yang penting ini, yaitu *Seminar Nasional Mahasiswa S3 Matematika se-Indonesia*. Kami menyambut baik acara yang diprakarsai dan diselenggarakan oleh mahasiswa S3 Matematika karena dapat mendorong terciptanya kerjasama keilmuan antara mahasiswa S3 Matematika/Statistika dan Pendidikan Matematika, atau antara mahasiswa S3 Matematika/Statistika/Pendidikan Matematika dengan para pakar Matematika/Statistika/Pendidikan Matematika lainnya di Indonesia.

Pada kesempatan ini, kami mengucapkan selamat mengikuti rangkaian *Seminar Nasional Mahasiswa S3 Matematika se-Indonesia* yang sangat penting ini. Semoga upaya yang kita laksanakan ini dapat menghasilkan sumbangan yang berharga dan menjadi bekal bagi kita semua dalam mencapai masyarakat Indonesia yang andal, cerdas, mempunyai komitmen moral dan semangat pengabdian.

Lebih jauh lagi, kami mengharapkan Seminar Nasional ini dapat memberikan wawasan dan berbagi informasi diantara mahasiswa Matematika/Statistika dan Pendidikan Matematika, atau Peminat Matematika tentang hasil penelitian yang telah atau sedang dilakukan.

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Yogyakarta, 31 Mei 2008

Pengelola S2/S3 Matematika FMIPA UGM

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# Chain Ladder Method as a Gold Standard to Estimate Loss Reserves

Aceng K. Mutaqin<sup>1)</sup>, Dumaria R. Tampubolon<sup>2)</sup>, Sutawanir Darwis<sup>2)</sup>

## Abstract

This paper review the chain ladder method to estimate loss reserves in general insurance and standard error of chain ladder reserve estimates that formulated by Mack (1993). The example to illustrate the method is also given. The authors provide a MATLAB program to implement the method.

**Key words:** *run-off triangle data, loss reserves, chain ladder method, MATLAB program*

## 1. Introduction

The correct estimation of the amount of money an insurance company should set aside to meet claims that arise in the future on written policies represents an important task for the insurance company. This estimation is often named as the loss reserve (or claim reserve). An insurance company needs to hold loss reserve because the timing of premiums receipt and claims payment do not coincide. There is the delay between the claim event and the claim settlement date. It could take a long time from the loss event to the reporting of the loss to the company, or it could take a long time from the day of reporting until the company knows the ultimate cost of the claim.

In the end of every reporting period (usually year and/or quarter) the insurance company should present in the accounting how much of the money that is allocated for this incurred but not settled loss. It is necessary for business planning, budgeting and product pricing (Olofsson, 2006).

The problem of how to calculate the loss reserve is usually solved with statistical methods. Since the amount and timing of future claims are unknown, this creates an uncertainty over the amount of reserves that is necessary. The degree of uncertainty depends on the class of business written, where we have short-tail and long-tail classes (Olofsson, 2006). A short-tail

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class is a business where the delay between the occurrence of a claim and the settlement is short, often less than a year, such as motor insurance that, for example, covers fire and theft (Olofsson, 2006). A long-tail business is a business where the delay between the occurrence of a claim and it being settle is long, more than couple of years, such as motor TPL (third party liability) that covers personal injury (Olofsson, 2006), and marine insurance (Hertig, 1985).

It is common to estimate loss reserve for long-tail business based on run-off triangle data (for example, see De Jong, 2006; England, dan Verrall, 2002; Verrall, 2002; Mack, 1993; Pinheiro, E Silva, dan Centeno, 2003; De Alba, 2006; Panning, 2004; dan Olofsson, 2006). Run-off triangle data provides a summary of underlying data set with individual claim figures (Antonio, Beirlant, Hoedemakers, dan Verlaak, 2006). There are different statistical methods to estimate loss reserve based on run-off triangle data. The chain ladder method is probably the most popular one for estimating loss reserve. The main reason for this is its simplicity and the fact that it is distribution-free. It is frequently used as a gold standard (benchmark) because of its generalized use an ease to apply. This method is deterministic and we only get a point estimate.

Is it well-known that chain ladder reserve estimates are very sensitive to variations in the data observed (Mack, 1993). Therefore it would be very helpful to know the standard error of the chain ladder reserve estimates as a measure of the uncertainty contained in the data. Mack (1993) developed a very simple formula and distribution-free for the standard error of chain ladder reserve estimates. This paper review the chain ladder method and standard error of chain ladder reserve estimates that formulated by Mack (1993).

The remainder of the paper is structured as follows. Section 2 presents the chain ladder method to estimate loss reserve. The standard error of the chain ladder reserve estimates that formulated by Mack (1993) is discussed in Section 3. Section 4 provides an example to illustrate the method. The implementation of the method in MATLAB program are given in Section 5.

## 2. Chain Ladder Method

Let  $C_{ij}$  denote the cumulative total claims amount of accident year  $i$ ,  $1 \leq i \leq n$ , paid up to development year  $j$ ,  $1 \leq j \leq n$ . The cumulative total claims amount  $C_{ij}$  have been observed for  $i + j \leq n + 1$  (run-off triangle data), the other amounts, especially the ultimate claims amount  $C_{in}$ ,  $i > 1$ , have to be predicted. The aim is to estimate the ultimate claims amount and the outstanding claims reserve

$$R_i = C_{in} - C_{i,n+1-i}; \text{ for accident year } i = 2, \dots, n \quad (1)$$

Run-off triangle data are presented in Table 1.

**Table 1** Run-off Triangle Data

| Accident Period | Development Period |             |          |          |          |             |          |
|-----------------|--------------------|-------------|----------|----------|----------|-------------|----------|
|                 | 1                  | 2           | ...      | $j$      | ...      | $n-1$       | $n$      |
| 1               | $C_{11}$           | $C_{12}$    | ...      | $C_{1j}$ | ...      | $C_{1,n-1}$ | $C_{1n}$ |
| 2               | $C_{21}$           | $C_{22}$    | ...      | $C_{2j}$ | ...      | $C_{2,n-1}$ |          |
| $\vdots$        | $\vdots$           | $\vdots$    | $\vdots$ | $\vdots$ | $\ddots$ |             |          |
| $i$             | $C_{i1}$           | $C_{i2}$    | ...      | $C_{ij}$ |          |             |          |
| $\vdots$        | $\vdots$           | $\vdots$    | $\ddots$ |          |          |             |          |
| $n-1$           | $C_{n-1,1}$        | $C_{n-1,2}$ |          |          |          |             |          |
| $n$             | $C_{n,1}$          |             |          |          |          |             |          |

The total reserve is

$$R = \sum_{i=2}^n R_i \quad (2)$$

The basic assumption of the chain ladder method is that there are development factors  $f_1, \dots, f_{n-1} > 0$  with  $E(C_{i,j+1} | C_{i1}, \dots, C_{ij}) = C_{ij}f_j$ ,  $1 \leq i \leq n$ ,  $1 \leq j \leq n-1$ .

The chain ladder method consists of estimating the  $f_k$  by

$$\hat{f}_j = \frac{\sum_{k=1}^{n-j} C_{k,j+1}}{\sum_{k=1}^{n-j} C_{k,j}}; \text{ for } 1 \leq j \leq n-1, \quad (3)$$

and the ultimate claims amount  $C_{in}$ , by

$$\hat{C}_{in} = C_{i,n+1-i} \hat{f}_{n+1-i} \cdots \hat{f}_{n-1} \quad (4)$$

or equivalently the reserve  $R_i$  by

$$\begin{aligned} \hat{R}_i &= \hat{C}_{in} - C_{i,n+1-i} \\ &= C_{i,n+1-i} (\hat{f}_{n+1-i} \cdots \hat{f}_{n-1} - 1) \end{aligned} \quad (5)$$

Because the chain ladder method does not take into account any dependencies between accident years, Mack (1993) added assumption that the variables  $C_{ij}$  of different accident years, i.e.  $\{C_{i1}, \dots, C_{in}\}$ ,  $\{C_{k1}, \dots, C_{kn}\}$ ,  $i \neq k$ , are independent. This must be regarded as a further implicit assumption of the chain ladder method.

Mack (1993) proved that

1.  $E(C_{in} | D) = C_{i,n+1-i} f_{n+1-i} \cdots f_{n-1}$ , where  $D = \{C_{ij} | i + j \leq n + 1\}$  is the set of all data observed so far.
2. The estimators  $\hat{f}_j$ , for  $1 \leq j \leq n - 1$ , are unbiased and uncorrelated of  $f_j$ .
3. The estimators  $\hat{C}_{in} = C_{i,n+1-i} \hat{f}_{n+1-i} \cdots \hat{f}_{n-1}$ , for  $2 \leq i \leq n$ , are unbiased of  $E(C_{in} | D) = C_{i,n+1-i} f_{n+1-i} \cdots f_{n-1}$ .
4. The reserve estimators  $\hat{R}_i = \hat{C}_{in} - C_{i,n+1-i}$ , for  $2 \leq i \leq n$ , are unbiased of the true reserves  $R_i = C_{in} - C_{i,n+1-i}$ .

### 3. Standard Error of Chain Ladder Reserve Estimates

The mean squared error of the estimator  $\hat{C}_{in}$  of  $C_{in}$  is defined to be

$$mse(\hat{C}_{in}) = E((\hat{C}_{in} - C_{in})^2 | D)$$

where  $D = \{C_{ij} | i + j \leq n + 1\}$  is the set of all data observed so far.

The mean squared error of the estimator  $\hat{R}_i$  of  $R_i$  is

$$\begin{aligned} mse(\hat{R}_i) &= E((\hat{R}_i - R_i)^2 | D) \\ &= E((\hat{C}_{in} - C_{in})^2 | D) \\ &= mse(\hat{C}_{in}) \\ &= Var(C_{in} | D) + (E(C_{in} | D) - \hat{C}_{in})^2 \end{aligned}$$

which shows that the mean squared error is the sum of the stochastic error (process variance) and of the estimation error.

Mack (1993) introduced an assumption that

$$\text{Var}(C_{i,j+1} | C_{i1}, \dots, C_{ij}) = C_{ij} \sigma_j^2, 1 \leq i \leq n, 1 \leq j \leq n-1,$$

with unknown parameters  $\sigma_j^2$ ,  $1 \leq j \leq n-1$ . This is the variance assumption which is implicitly underlying the chain ladder method.

The parameters  $\sigma_j^2$ ,  $1 \leq j \leq n-1$ , are estimated by

$$\hat{\sigma}_j^2 = \frac{1}{n-j-1} \sum_{i=1}^{n-j} C_{ij} \left( \frac{C_{i,j+1}}{C_{ij}} - \hat{f}_j \right)^2, 1 \leq j \leq n-2. \quad (6)$$

If  $\hat{f}_{n-1} = 1$  and if the claims development is believed to be finished after  $n-1$  years, then  $\hat{\sigma}_{n-1} = 0$ , otherwise

$$\hat{\sigma}_{n-1} = \min(\hat{\sigma}_{n-2}^4 / \hat{\sigma}_{n-3}^2, \min(\hat{\sigma}_{n-3}^2, \hat{\sigma}_{n-2}^2)) \quad (7)$$

Mack (1993) proved that

1. The mean squared error  $mse(\hat{R}_i)$  can be estimated by

$$\widehat{mse}(\hat{R}_i) = \hat{C}_{in}^2 \sum_{j=n+1-i}^{n-1} \frac{\hat{\sigma}_j^2}{\hat{f}_j^2} \left( \frac{1}{\hat{C}_{ij}} - \frac{1}{\sum_{k=1}^{n-j} C_{kj}} \right) \quad (8)$$

where  $\hat{C}_{ij} = C_{i,n+1-i} \hat{f}_{n+1-i} \dots \hat{f}_{j-1}$ ,  $i+j > n+1$ , are the estimated values of the future  $C_{ij}$  and  $\hat{C}_{i,n+1-i} = C_{i,n+1-i}$ .

2. The mean squared error of the total reserve estimate  $\hat{R}$  can be estimated by

$$\widehat{mse}(\hat{R}) = \sum_{i=2}^n \left\{ [se(\hat{R}_i)]^2 + \hat{C}_{in} \left[ \sum_{k=i+1}^n \hat{C}_{kn} \right] \sum_{j=n+1-i}^{n-1} \frac{2\hat{\sigma}_j^2 / \hat{f}_j^2}{\sum_{m=1}^{n-j} C_{mj}} \right\} \quad (9)$$

where  $se(\hat{R}_i)$  is the standard error of  $\hat{R}_i$ , i.e. the squared root of  $mse(\hat{R}_i)$ .

#### 4. Example

In this paper, we use the data which were used by Mack (1993) in the first example to apply the chain ladder reserve estimates and its standard errors. The data are presented in Table 2.

**Table 2** Run-off Triangle Data (Cumulative Figures)

| $i$ | $C_{i1}$ | $C_{i2}$ | $C_{i3}$ | $C_{i4}$ | $C_{i5}$ | $C_{i6}$ | $C_{i7}$ | $C_{i8}$ | $C_{i9}$ | $C_{i10}$ |
|-----|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|
| 1   | 357848   | 1124788  | 1735330  | 2218270  | 2745596  | 3319994  | 3466336  | 3606286  | 3833515  | 3901463   |
| 2   | 352118   | 1236139  | 2170033  | 3353322  | 3799067  | 4120063  | 4647867  | 4914039  | 5339085  |           |
| 3   | 290507   | 1292306  | 2218525  | 3235179  | 3985995  | 4132918  | 4628910  | 4909315  |          |           |
| 4   | 310608   | 1418858  | 2195047  | 3757447  | 4029929  | 4381982  | 4588268  |          |          |           |
| 5   | 443160   | 1136350  | 2128333  | 2897821  | 3402672  | 3873311  |          |          |          |           |
| 6   | 396132   | 1333217  | 2180715  | 2985752  | 3691712  |          |          |          |          |           |
| 7   | 440832   | 1288463  | 2419861  | 3483130  |          |          |          |          |          |           |
| 8   | 359480   | 1421128  | 2864498  |          |          |          |          |          |          |           |
| 9   | 376686   | 1363294  |          |          |          |          |          |          |          |           |
| 10  | 344014   |          |          |          |          |          |          |          |          |           |

This yields the parameter estimates  $\hat{f}_j$  and  $\hat{\sigma}_j^2$ , ( $j = 1, 2, \dots, 9$ ):

**Table 3** The Parameter Estimates

|                    | $j=1$   | $j=2$    | $j=3$   | $j=4$   | $j=5$   | $j=6$   | $j=7$   | $j=8$   | $j=9$   |
|--------------------|---------|----------|---------|---------|---------|---------|---------|---------|---------|
| $\hat{f}_j$        | 3.49061 | 1.74733  | 1.45741 | 1.17385 | 1.10382 | 1.08627 | 1.05387 | 1.07656 | 1.01772 |
| $\hat{\sigma}_j^2$ | 160280  | 37736.86 | 41965.2 | 15182.9 | 13731.3 | 8185.77 | 446.617 | 1147.37 | 446.617 |

Table 4 shows the ultimate claims, the estimated reserves, the standard errors of the estimated reserves, and the standard errors of the estimated reserves in % of  $\hat{R}_i$  for  $i = 2, 3, \dots, 10$ . The last row in Table 4 contains the estimated total reserve and its standard error.

**Table 4** Summary of The Chain Ladder Reserve Estimates

|                | Ultimate Claims | Estimated Reserves | Standard Errors | Standard Errors in % of $\hat{R}_i$ |
|----------------|-----------------|--------------------|-----------------|-------------------------------------|
| $i=2$          | 5433719         | 94634              | 75535           | 0.80                                |
| $i=3$          | 5378826         | 469511             | 121699          | 0.26                                |
| $i=4$          | 5297906         | 709638             | 133549          | 0.19                                |
| $i=5$          | 4858200         | 984889             | 261406          | 0.27                                |
| $i=6$          | 5111171         | 1419459            | 411010          | 0.29                                |
| $i=7$          | 5660771         | 2177641            | 558317          | 0.26                                |
| $i=8$          | 6784799         | 3920301            | 875328          | 0.22                                |
| $i=9$          | 5642266         | 4278972            | 971258          | 0.23                                |
| $i=10$         | 4969825         | 4625811            | 1363155         | 0.29                                |
| <b>Overall</b> |                 | 18680856           | 2447095         | 0.13                                |

```
function out=CLM(C)
% This program is designed to implement the chain ladder method.
% The input program is C : a matrix contains the run-off triangle data
% in cumulative form
% The output program is out : a matrix contains
% column 1: Ultimate claims
% column 2: Reserve estimates
% (including total reserve estimate in the last row)
% column 3: Standard errors of reserve estimates
% (including for total reserve estimate in the last row)
% column 4: Standard errors in % of reserve estimates
% (column 3/column 2)
n=size(C,1); % number of accident years/development years
m=n-1;
for j=1:m
    f(j,1)=sum(C(:,j+1))/(sum(C(:,j))-C(n+1-j,j)); % development factors
end
C1=C;
k=n+1;
for i=2:n
    k=k-1;
    for j=k:n
        C1(i,j)=C1(i,j-1)*f(j-1,1);
    end
    out(i,1)=C1(i,n); % Ultimate Claims
    out(i,2)=C1(i,n)-C1(i,k-1); % Reserve Estimates
end
out(n+1,2)=sum(out(2:n,2)); % Total Reserve Estimate
m2=n-2;
for j=1:m2
    sigma2(j,1)=sum(C(1:n-j,j).*(C(1:n-j,j+1)-C(1:n-j,j)-f(j,1)).^2)/(n-j-1);
end
if f(n-1,1)==1
    sigma2(m2+1,1)=0;
else
    a=(sigma2(n-2,1))^2/sigma2(n-3,1);
    b=min(sigma2(n-3,1),sigma2(n-2,1));
    sigma2(m2+1,1)=min(a,b);
end
for i=2:n
    k1=n+1-i;
    k2=n-1;
    s1=0;
    for k=k1:k2
        b=sum(C1(1:n-k,k));
        s1=s1+sigma2(k,1)/f(k,1)^2*(1/C1(i,k)+1/b);
    end
    out(i,3)=sqrt(C1(i,n)^2*s1); % Standard Errors of Reserve Estimates
end
s2=0;
for i=2:n
```

```
k1=n+1-i;k2=n-1;
a1=sum(C1(i+1:n,n));
a2=0;
for k=k1:k2
    b=sum(C1(1:n-k,k));
    a2=a2+2*sigma2(k,1)/f(k,1)^2/b;
end
s2=s2+out(i,3)^2+C1(i,n)*a1*a2;
end
out(n+1,3)=sqrt(s2); % Standard Errors of Total Reserve Estimates
for i=2:n+1
    out(i,4)=out(i,3)/out(i,2); % Standard Errors in % of Reserve Estimates
end
```

**Figure 1**  
MATLAB Program to Implement The Chain Ladder Method

## 5. Implementing the Chain Ladder Method in MATLAB

The chain ladder method can be implemented using MATLAB program (see Figure 1). This program can be used to calculate the ultimate claims, the estimated reserves, the standard errors of the estimated reserves, and the standard errors of the estimated reserves in % of the estimated reserves. The values in Table 4 are generated using the MATLAB program in Figure 1.

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