Konferensi Kebangsaan
PEMODELAN MATEMATIK & STATISTIK

PROSIDING

PEMODELAN MATEMATIK & STATISTIK DALAM PEMBANGUNAN NEGARA

Editor
Adem Kilicman • Ural Bekbaev • Zainidin K. Eshkuvatov • Norazak Senu

Anjuran

UPM
UNIVERSITI PUTRA MALAYSIA

- Jabatan Matematik, Fakulti Sains, Universiti Putra Malaysia
- 5-6 September 2006
- Dewan Persidangan IDEAL, Universiti Putra Malaysia

Perpustakaan Negara Malaysia Data Pengkatalogan-dalam-Penerbitan

Konferensi Kebangsaan Pemodelan Matematik dan Statistik (2006 : Serdang)
"Pemodelan matematik dan statistik dalam pembangunan negara":
prosiding Konfrensi Kebangsaan Pemodelan Matematik dan Statistik,
5-6 September 2006, Dewan Persidangan IDEAL, UPM / di sunting oleh
Adem Kilicman...[et al.] ; anjuran: Jabatan Matematik, Fakulti Sains, UPM.
ISBN 978-983-3455-68-3
1. Mathematical models--Congresses. 2. Linear models (Statistics)--Data
processing--Congresses. I. Kilicman, Adem. II. Universiti Putra Malaysia.
Jabatan Matematik. III. Judul.
006.6511.8

Dicetak oleh
Penerbit
Universiti Putra Malaysia
Serdang, Selangor.

KANDUNGAN

Konferensi Kebangsaan Pemodelan Matematik & Statistik Sempena 75 tahun UPM
Jabatan Matematik, UPM 5-6 September 2006

MEDIAN SURVIVAL TIME OF WEIBULL DISTRIBUTION

Abdul Kudus[1,3] *and Noor Akma Ibrahim*[1,2]
[1]*Institute for Mathematical Research, University Putra Malaysia*
[2]*Department of Mathematics, Faculty of Science, University Putra Malaysia*
[3]*Department of Statistics, Universitas Islam Bandung, Indonesia*

Abstrak

In many applications of lifetime data analysis, it is important to perform inferences about the median of the distribution function in situations of lifetime data modeling with skewed distribution. For lifetime distributions where the median of the distribution function can be analytically calculated, its maximum likelihood estimator is easily obtained from the invariance properties of the maximum likelihood estimators. From the asymptotical normality of the maximum likelihood estimators, confidence intervals can be obtained. However, these results might not be very accurate for small sample sizes and/or large proportion of censored observations. Considering the Weibull distribution for the lifetime data, we present and compare the accuracy of asymptotical confidence intervals with two confidence intervals based on bootstrap simulation. The alternative methodology of confidence intervals for the median of the Weibull distribution function is illustrated by using real data examples. The nonparametric bootstrap procedure was implemented in the SAS® system which incorporated *proc nlp*, *proc surveyselect* and *proc iml* in the SAS® *macro* environment.

Key word: Median, Weibull, Bootstrap

1. Introduction

In lifetime data analysis, we usually have a skewed distribution function. One of the skewed distributions which plays a central role in the analysis of survival data is Weibull distribution, introduced by W. Weibull in 1951 in the context of industrial reliability testing. Indeed, this distribution is as central to the parametric analysis of survival data as the normal distribution is in linear modeling. For the skewed distribution, a more appropriate and more tractable summary of the location of the distribution is the median survival time [1].

Usually, we have interest in the estimation of the median survival time where the central of tendency of the distribution function occurs.

Considering the Weibull distribution, we introduce asymptotical based inferences and bootstrap based inferences for the median of the survival time. It is important to note that usually, in the literature of lifetime data analysis, confidence intervals for the median of the survival time are based on asymptotic arguments. A recent study about the Weibull distribution, related to this work, is presented in [2].

The paper is organized as follows: in Section 2 we introduce some characteristics of the Weibull distribution; in Section 3 we introduce the likelihood function in the presence of censored observations; in Section 4 we have some comparisons between asymptotical based inferences and bootstrap simulation based inferences for the median survival time; in Section 5 we give illustrative example with real data set. A SAS macro to obtain asymptotical and boot strap confidence intervals is briefly discussed in Section 6.

2. The Weibull distribution

Let T be a random variable representing the lifetime of a unit or patient with a Weibull distribution with hazard function given by

$$h(t) = \lambda \gamma t^{\gamma-1}, \; 0 \le t < \infty \qquad (1)$$

This function depends on two parameters λ and γ, which are both greater than zero. In the particular case where $\gamma = 1$, the hazard function takes a constant value λ, and the survival times have an exponential distribution. For other value of γ, the hazard function increases or decreases monotonically, that is, it does not change direction. The shape of the hazard function depends critically on the value of γ, and so γ is known as the shape parameter, while the parameter λ is a scale parameter. The general form of this hazard function for different values of γ is shown in fig. 1. For this particular choice of hazard function, the survivor function [3] is given by

$$S(t) = \exp\left\{-\int_0^t \lambda\gamma u^{\gamma-1} du\right\} = \exp(-\lambda t^{\gamma}) \qquad (2)$$

The corresponding probability density function is then

$$f(t) = \lambda\gamma t^{\gamma-1} \exp(-\lambda t^{\gamma}) \qquad (3)$$

for $0 \leq t < \infty$, which is the density of a random variable that has a Weibull distribution with scale parameter λ and shape parameter γ. The right-hand tail of this distribution is longer than the left-hand one, and so the distribution is positively skewed.

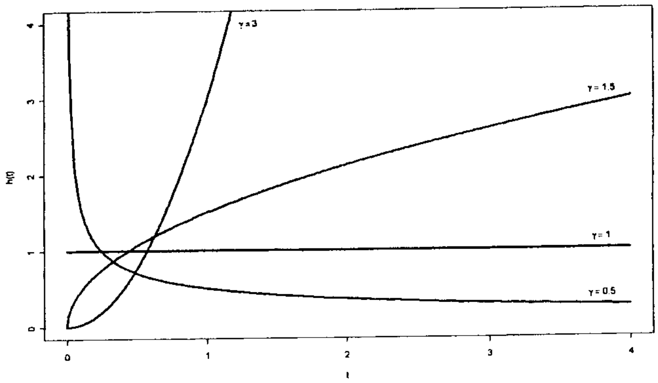

Fig. 1. Weibull hazard function for λ = 1 and γ = 0.5, 1.0, 1.5 and 3.0

The mean, or expected value, of a random variable T that has Weibull distribution is as follows

$$E(T) = \lambda^{-1/\gamma}\Gamma(\gamma^{-1} + 1) \qquad (4)$$

where $\Gamma(x)$ is the gamma function defined by

$$\Gamma(x) = \int_0^{\infty} u^{x-1} e^{-u} du \qquad (5)$$

However, since the Weibull distribution is skewed, a more appropriate and more tractable summary of the location of the distribution is the median survival time. This is the value $t(50)$ such that $S\{t(50)\} = 0.5$, so that

$$\exp\{-\lambda[t(50)]^{\gamma}\} = 0.5$$

and

$$t(50) = \left[\frac{1}{\lambda}\log 2\right]^{1/\gamma} \qquad (6)$$

Since the Weibull hazard function can take a variety of forms depending on the value of the shape parameter γ, appropriate summary statistics can be easily obtained. This distribution is widely used in the parametric analysis of survival data.

3. The likelihood function in the presence of right censored data

Let T_1^0, ..., T_n^0 be the true lifetimes of a sample of size n, assumed to be independent identically distributed with a Weibull distribution with hazard function (1). Assuming that the observations are subject to arbitrary right censoring, the period of follow-up for the ith individual is limited to a value C_i. Then, the observed survival time of the ith individual is given by $t_i = \min(T_i^o, C_i)$.

Define δ_i such that $\delta_i = 0$ if $T_i^o \geq C_i$ (a censored observation) and $\delta_i = 1$ if $T_i^o < C_i$ (an observed death or failure of some kind). The likelihood function for λ and γ is given by

$$L(\lambda, \gamma \mid t) = \prod_{i=1}^{n} \{\lambda \gamma t_i^{\gamma-1} \exp(-\lambda t_i^{\gamma})\}^{\delta_i} \{\exp(-\lambda t_i^{\gamma})\}^{1-\delta_i}$$

The corresponding log-likelihood function is given by

$$l(\lambda, \gamma \mid t) = \sum_{i=1}^{n} \delta_i \log(\lambda\gamma) + (\gamma - 1)\sum_{i=1}^{n} \delta_i \log t_i - \lambda \sum_{i=1}^{n} t_i^{\gamma} \quad (7)$$

Maximum likelihood estimators for λ and γ are obtained by differentiating this function with respect to λ and γ, equating the derivative to zero and solve these equations

$$\begin{aligned} &\frac{1}{\lambda}\sum_{i=1}^{n} \delta_i - \sum_{i=1}^{n} t_i^{\gamma} = 0 \\ &\frac{1}{\gamma}\sum_{i=1}^{n} \delta_i + \sum_{i=1}^{n} \delta_i \log t_i - \lambda \sum_{i=1}^{n} t_i^{\gamma} \log t_i = 0 \end{aligned} \quad (8)$$

The maximum likelihood estimator for the median survival time $t(50)$ is obtained from the maximum likelihood estimators $\hat{\lambda}$ and $\hat{\gamma}$, that is

$$\hat{t(50)} = \left[\frac{1}{\hat{\lambda}} \log 2\right]^{1/\hat{\gamma}} \quad (9)$$

Asymptotical confidence intervals for $t(50) = g(\lambda,\gamma)$ are obtained using the delta method, that is, $\hat{t(50)} \overset{a}{\sim} N[t(50), \mathrm{var}(t(50))]$. Under the delta method, the asymptotical variance of $\hat{t(50)} = g(\hat{\lambda}, \hat{\gamma})$ is given by

$$\begin{aligned} Var[g(\hat{\lambda},\hat{\gamma})] = &\left[\frac{\partial}{\partial\lambda} g(\hat{\lambda},\hat{\gamma})\right]^2 Var(\hat{\lambda}) + \left[\frac{\partial}{\partial\gamma} g(\hat{\lambda},\hat{\gamma})\right]^2 Var(\hat{\gamma}) \\ &+ 2\left[\frac{\partial}{\partial\lambda} g(\hat{\lambda},\hat{\gamma})\right]\left[\frac{\partial}{\partial\gamma} g(\hat{\lambda},\hat{\gamma})\right] Cov(\hat{\lambda},\hat{\gamma}) \end{aligned} \quad (10)$$

where

$$\frac{\partial}{\partial\lambda} g(\hat{\lambda},\hat{\gamma}) = -\left.\frac{t(50)}{\lambda\gamma}\right|_{\lambda=\hat{\lambda},\gamma=\hat{\gamma}}$$

$$\frac{\partial}{\partial\gamma} g(\hat{\lambda},\hat{\gamma}) = -\left.\frac{t(50)^{\gamma+1}}{\gamma^2}\right|_{\lambda=\hat{\lambda},\gamma=\hat{\gamma}}$$

and the asymptotical variances and covariance for $\hat{\lambda}$ and $\hat{\gamma}$ are obtained from the inverse of the Fisher information matrix for λ and γ.

Alternatively to the asymptotical based confidence intervals for $t(50)$, we could use nonparametric bootstrap simulation methods by resampling with replacement the available data $(t_1, \delta_1), \ldots, (t_n, \delta_n)$ [4].

4. Bootstrap confidence intervals for *t*(50)

In this section we introduce the steps for the construction of bootstrap confidence intervals for $t(50)$, the median of the Weibull distribution function. The advantage of the bootstrap is that the joint distribution of the maximum likelihood estimators is not assumed to be normal, unlike in the delta method.

We consider two bootstrap methods to construct the confidence intervals for $t(50)$: the *p*-

Bootstrap method suggested by Efron [5], based on the percentiles of the bootstrap distribution, and the t-Bootstrap method suggested by Hall [6]. Other existing alternatives for the p-Bootstrap and the t-Bootstrap, not considered in this paper, also could be used to construct confidence intervals. For a complete review of available approaches to bootstrap confidence intervals see [5] and [7].

Let $U = (\boldsymbol{t},\boldsymbol{\delta})$ be the observed data where $\boldsymbol{t} = (t_1, \ldots, t_n)$ is the vector of lifetime data and $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_n)$ is the vector of indicators of censored observations.

4.1. p-Bootstrap

[a] Random select, with replacement from U, a bootstrap sample $(t_1^*, \delta_1^*), \ldots, (t_n^*, \delta_n^*)$.

[b] From the bootstrap sample in [a], find the maximum likelihood estimates of $t(50)$, denoted by $\hat{t(50)}^*$.

[c] Repeat steps [a] and [b], B times.

[d] From $\hat{t(50)}^* = \left(\hat{t(50)}^*_{(1)} \leq \hat{t(50)}^*_{(2)} \leq \ldots \leq \hat{t(50)}^*_{(B)}\right)$ find a $100\times(1-\alpha)\%$ bootstrap confidence interval given by $\left(\hat{t(50)}^*_{(q_1)}, \hat{t(50)}^*_{(q_2)}\right)$ where $q_1 = [(\alpha/2)B]$ and $q_2 = B - q_1$.

4.2. t-Bootstrap

[a']Random select, with replacement from U, a bootstrap sample $(t_1^*, \delta_1^*), \ldots, (t_n^*, \delta_n^*)$.

[b']From the bootstrap sample in [a'], find the maximum likelihood estimates of $t(50)$, denoted by $\hat{t(50)}^*$.

[c']Repeat steps [a'] and [b'], B times.

[d']Same as in [d] above, find $T^* = (T^*_{(1)}, \ldots, T^*_{(B)})$, where $T^*_{(i)} \leq T^*_{(j)}$ for $i, j = 1, \ldots, B$; $i \neq j$ given by

$$T_i^* = \frac{\left(\hat{t(50)}_i^* - \hat{t(50)}\right)}{\hat{\sigma}_i^*} \quad (11)$$

where $\hat{t(50)}$ is the maximum likelihood estimates for $t(50)$ and $\hat{\sigma}_i^*$ is the standard error of $\hat{t(50)}_i^*$. Since $\hat{\sigma}_i^*$ $(i = 1, \ldots, B)$ can be calculated directly by the inverse of the Fisher information matrix, it is not necessary to resample from the bootstrap sample [4] and [8].

[e']From T^* we find a $100 \times (1-\alpha)\%$ bootstrap confidence interval for $t(50)$ given by

$$\left(\hat{t(50)} - \hat{\sigma}T^*_{(q_2)}; \hat{t(50)} - \hat{\sigma}T^*_{(q_1)}\right) \quad (12)$$

where q_1 and q_2 are defined in [d] and $\hat{\sigma} = \sqrt{Var\left(\hat{t(50)}\right)}$, (and $\hat{t(50)}$ and $\hat{\sigma}$ are calculated from the original lifetime data).

5. An example with a real data set

As an example, consider the sample of strike lengths in days given in [9]. The data pertain to U.S. manufacturing industries for the period 1968 through 1976 and cover official strikes involving 1,000 workers or more. The complete and censored data sets considered are as in [10] and [2]. The restriction is applied to only strikes beginning in June of each year for a total number of 62 observations, which 12 of them are censored due to duration greater than or equal to 80 days. In Tables 1 and 2, we have 95% asymptotical and bootstrap confidence interval for parameters $t(50)$ and γ. The empirical bootstrap distributions are presented in

Fig.2

Table 1. Maximum likelihood estimates and asymptotical confidence intervals.

Parameter	MLE	SE	95% Confidence Interval
t(50)	27.6356	4.7158	(18.3927; 36.8785)
γ	0.8857	0.1061	(0.6777; 1.0936)

To check if the normality of the empirical bootstrap distributions for $t(50)$ and γ are appropriate, we have in Fig. 2(bottom row), their normal quantile-quantile plots. If we have normality, then the points in these plots should lie roughly on a straight line. From these plots, we clearly observe that the normality assumption is not appropriate, which justifies the use of bootstrap methods to construct confidence intervals for the parameters.

Table 2. Bootstrap estimates, p-Bootstrap and t-Bootstrap confidence intervals.

Parameter	MLE[a]	SE[a]	95% Confidence Interval	
			p-Bootstrap	*t*-Bootstrap
t(50)	27.9261	3.5120	(21.6245;35.1736)	(22.7818;33.7460)
γ	0.8949	0.0763	(0.7555;1.0616)	(0.7696;1.0031)

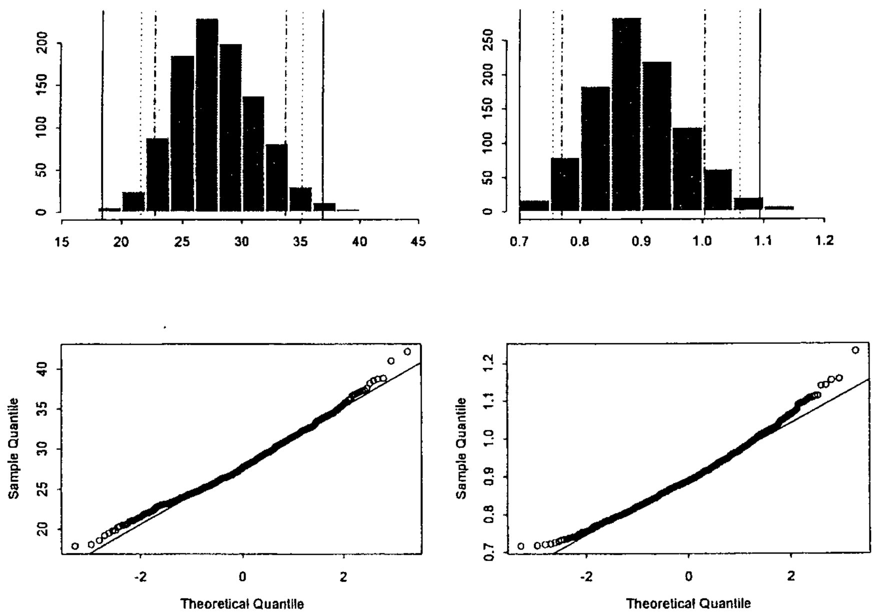

Fig. 2. (Top row) Distribution of B = 1000 bootstrap replications for parameter t(50) and γ along with their 95% confidence intervals, where (—): asymptotic confidence interval, (...): p-Bootstrap and (-.-.): t-Bootstrap confidence intervals, (Bottom row) quantile-quantile plots for t(50) and γ.

From the obtained results of Table 3, we observed that the obtained bootstrap confidence intervals are more accurate than the obtained asymptotical intervals, even in the situation of a large sample size (n = 62) and few censored observations.

Table 3. Range (R) and asymmetry index (F) for the 95% CI for t(50) and γ

Interval	Parameter			
	$t(50)$		γ	
	R	F	R	F
Asymptotical	18.4858	1.0000	0.4158	1.0000
p-Bootstrap	13.5492	1.2540	0.3062	1.3515
t-Bootstrap	10.9641	1.2589	0.2335	1.0120

6. A SAS macro for asymptotic and bootstrap confidence intervals

A SAS macro has been written to implement the asymptotic and bootstrap confidence intervals presented in numerical examples, Section 5. In the macro code, we do the resampling using *proc surveyselect* and the maximum likelihood estimates for $t(50)$ and γ are obtained via the *proc nlp* using the trust-region method [11]. From the B bootstrap estimates for $t(50)$, γ, Var($t(50)$) and Var(γ), SAS/IML is used to provide asymptotic and bootstrap confidence intervals estimates. The SAS macro, along with instructions on its use, is provided in appendix.

7. Conclusions

Considering the Weibull distribution, we presented two bootstrap based methods to construct confidence intervals for the median of survival time. We have showed in numerical examples that the nonparametric bootstrap can be quite useful. Even for large sample sizes and small proportion of censored data, we observed better inference results considering bootstrap based methods in comparison to the usual asymptotical inference based on the normality of the maximum likelihood estimators. As pointed in [2], the difference between asymptotic confidence interval and the bootstrap confidence interval might be due to the fact that the variance obtained by the delta method depends on Taylor series approximation where the error terms are ignored. These terms are part of the variances computed by bootstrapping.

The bootstrap can also be used to obtain confidence intervals for other functions of the parameters. For example, we could obtain confidence intervals for the mean of the Weibull distribution.

The nonparametric bootstrap procedure was implemented in the SAS system and the macro code is readily available in the appendix.

Acknowledgements

This work is funded by IRPA Grant vote no. 54411 from the Ministry of Science, Technology and Innovation, Malaysia.

References

[1] Collett, D. 1994. *Modelling Survival Data in Medical Research,* Chapman & Hall, New York.

[2] Rekkas, M. and Wong, A. 2005. Third-order inference for the Weibull distribution. *Computational Statistics and Data Analysis*. 49:499-525.

[3] Lawless, J.F. 2003. *Statistical Models and Methods for Lifetime Data*, 2nd Ed. Wiley Interscience, New York.

[4] Efron, B. and Tibshirani, R.J. 1993. *An introduction to the bootstrap*, vol. 57 of Monographs on Statistics and Applied Probability, Chapman & Hall, New York.

[5] Efron, B. 1982. The jackknife, the bootstrap and other resampling plans, vol. 38 of *CBMS-NSF Regional Conference Series in Applied Mathematics, Society for Industrial and Applied Mathematics (SIAM)*, Philadelphia, PA.

[6] Hall, P. 1988. Theoretical comparison of bootstrap confidence intervals, *Ann. Stat.* 16 (3) 927—985.

[7] Davison, A.C. and Hinkley, D.V. 1997. *Bootstrap methods and their application*, Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge University Press, Cambridge.

[8] Carpenter, J. and Bithell, J. 2000. Bootstrap confidence intervals when, which, what? A practical guide for medical statistician, *Stat. Med.* 19:1141—1164.

[9] Kennan, J. 1985. The duration of contract strikes in U.S. manufacturing. *J. Econom.* 28, 5–28.

[10] Kiefer, N. 1988. Economic duration data and hazard functions. *J. Econom. Literature* 26, 646–679.

[11] Coleman, T.F. and Hempel, C. 1990. Computing a trust region step for a penalty function, SIAM *J. Sci. Stat. Comput.* 11 (1):180—201.

Appendix. SAS Macro for asymptotic and bootstrap confidence interval

```
data strike;
 input time delta @@;
 datalines;
      7    1    9    1    13   1    14   1    26   1    29   1
      52   1    80   0    9    1    37   1    41   1    49   1
      52   1    80   0    3    1    17   1    19   1    28   1
      72   1    80   0    80   0    80   0    80   0    80   0
      80   0    15   1    61   1    80   0    2    1    25   1
      80   0    3    1    10   1    1    1    2    1    2    1
      3    1    3    1    4    1    8    1    11   1    22   1
      23   1    27   1    32   1    33   1    35   1    43   1
      43   1    44   1    80   0    5    1    49   1    2    1
      12   1    12   1    21   1    21   1    27   1    38   1
      42   1    80   0;
 %macro names(in=,start=1,end=);
  %do i=&start %to &end;
   &in&i
  %end;
 %mend names;
 /* Maximum Likelihood Estimation from Original Survival Times */
 proc nlp data = strike tech = tr cov=2 vardef=n pcov noprint
```

```
   outest=mles(keep=t50 gamma _type_
   where=(_type_ in ("PARMS","COV2: H","LOWERBD")));
   max loglik;
   parms t50 = 27.27310, gamma = 0.921;
   bounds t50 > 0, gamma > 0;
        lambda = log(2)/(t50**gamma);
        term1 = log(lambda)+log(gamma)+(gamma-1)*log(time);
        term2 = -lambda*time**gamma;
        loglik = delta*term1+term2;
run;
%macro takesampleandestimate(repetition=,inpt=,seed=,size=);
        proc surveyselect data = &inpt
                   out  = outp
                   seed = &seed
                   method = urs
                   n = &size noprint;
        run;
        proc nlp data = outp tech=tr cov=2
                vardef=n pcov noprint inest=mles
                outest=aux&repetition(keep=t50 gamma _type_
                where=(_type_ in ("PARMS","COV2: H")));
        max loglik;
        parms t50, gamma;
                lambda = log(2)/(t50**gamma);
                term1 = log(lambda)+log(gamma)+(gamma-1)*log(time);
                term2 = -lambda*time**gamma;
                loglik = delta*term1+term2;
        run;
%mend takesampleandestimate;
%macro bootstrap(rept=, alpha=, inpt=, sizesample=, seed=);
        %do i = 1 %to &rept;
                %takesampleandestimate(repetition=&i,
                                        inpt=&inpt,
                                        seed=&seed,
                                        size=&sizesample);
                %put simulation &i from &rept;
                %let seed = %eval(&seed+1);
```

```
data auxl(drop=_type_);
        set %names(in=aux,start=1,end=&rept);
run;
proc iml; reset noname;
    aux = &rept;
    namesc = {"MLE" "SE" "LOWER BOUND" "UPPER BOUND"};
    namescboot = {"BootEst" "BootSE" "LOWER BOUND" "UPPER
        BOUND"};
    namesr = {"t50", "gamma"};
    namers = {"range" "shape"};
    start order(inpt);
        nc = ncol(inpt);
        aux1 = inpt;
        do i=1 to nc;
                aux2 = rank(aux1[,i]);
                aux1[aux2,i] = inpt[,i];
        end;
        return(aux1);
    finish order;
    use auxl; read all into mat;
    boots = shape(mat,aux,6)[,{1 2 3 6 5}];
    sb = sqrt(boots[,{3 4}]);
    use mles; read all into mles;
    mle = mles[{1 2}];
    se = sqrt(mles[{5 8}]);
    q1 = aux*(&alpha/2);
    q2 = aux - q1;
    pct = mle || se || (mle-probit(1-&alpha/2)*se)||    (mle-probit(&alpha/2)*se);
    print "Asymptotic Confidence Interval 100*(1-
            &alpha/2)%",
    pct[colname=namesc rowname=namesr f=.4];
    range = pct[,4] - pct[,3];
    shape = (pct[,4] - mle)/(mle - pct[,3]);
    print(range||shape)[colname=namers rowname=namesr f=.4];   bootpar =
boots[,{1 2}];
    bootest = t(bootpar[:,]);
```

```
n = nrow(boots);
bootcorr = bootpar-shape(bootpar[:,],n,2);
bootse = t(sqrt(bootcorr[##,]/(n-1)));
pct = bootest||bootse||t(order(boots[,{1 2}])[q1//q2,]);    print "p-Bootstrap
Confidence Interval 100*(1-
    alpha/2)%",
pct[colname=namescboot rowname=namesr f=.4];range = pct[,4] - pct[,3];
shape = (pct[,4] - mle)/(mle - pct[,3]);
print (range||shape)[colname=namers rowname=namesr
    f=.4];
T =t(order((boots[,{1 2}]
    -shape(mle,aux,2))/sb)[q1//q2,]);
pct = bootest||bootse||(mle-T[,2]#se)||(mle-T[,1]#se);    print "t-Bootstrap
Confidence Interval 100*(1
    -alpha/2)%",
pct[colname=namescboot rowname=namesr f=.4];      range = pct[,4] - pct[,3];
shape = (pct[,4] - mle)/(mle - pct[,3]);
print (shape||shape)[colname=namers rowname=namesr
    f=.4];
/* sending out bootpar */
create databoot from bootpar[colname={'t50' 'gamma'}];        append from
bootpar;
quit;
proc gchart data=databoot;
  vbar t50 / midpoints = 15 TO 40 BY 2.5;
  vbar gamma/ midpoints = 0.55 To 1.55 BY 0.1;
run;
proc export data=databoot
        outfile='c:\bootresult.xls'
        dbms=excel
        replace;
run;
%mend bootstrap;
/* Run the macro */
%bootstrap(rept=1000,alpha=0.05,inpt=strike,sizesample=62,seed=123);
```